Dynamical Gauge Symmetry Breaking on Compactified Space

Tatsu MISUMI BNL

Kashiwa, TM, [arXiv:1302.2196].
Kouno, TM, Kashiwa, Makiyama, Sasaki, Yahiro, in preparation.

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Dynamical Gauge Symmetry Breaking - DGSB

GUT ?

$\downarrow$ DGBS at GUT scale

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$\downarrow$ DGBS at EW scale

$\boxed{SU(3)_C} \times \boxed{U(1)_{EM}}$

$\downarrow$ CSC $\quad \downarrow$ SC

DGBS at extreme cond.

Understanding of DGSB : Key in Modern Physics Frontier

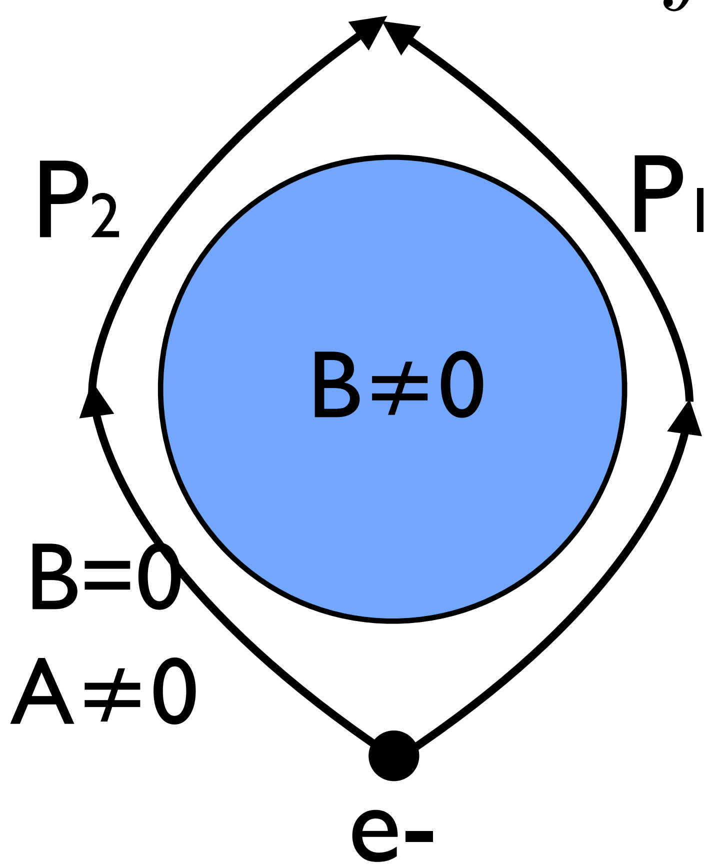
CW mechanism, Technicolor, Gauge-Higgs.....

Gauge Symmetry breaking by Non-Abelian AB effect

U(1) Aharonov-Bohm Effect

$$\delta\phi = \frac{e}{\hbar}\int_C A dx \longleftrightarrow \text{Wilson-loop phase}$$

$$e^{i\delta\phi} \sim W = P\exp(ig\int_C A dx)$$



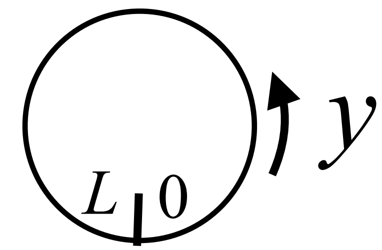
- Even if no field strength(x-indep A),
  AB phase affects physics.

- Gauge-invariant quantity.
  Cannot be gauged away. It is Physics!

Purely quantum phenomenon!

SU(N) gauge theory on $R^d \times S^1$.

BC: $$\begin{aligned} A(x, y+L) &= U A(x,y) U^\dagger \\ \psi(x, y+L) &= e^{i\beta} U \psi(x,y) \end{aligned}$$



- Wilson loop in compacted direction $W = P\exp\left\{ ig \int_C dy A_y \right\}$

1. constant eigenvalues $\mathrm{diag}[e^{2\pi i q_1}, e^{2\pi i q_2}, \cdots, e^{2\pi i q_N}]$
2. invariant under gauge transformation keeping B.C.
3. cannot be gauged away, and contributes to physics

→ Non-abelian AB effect

cf.) Effective longitudinal gluon mass in Finite-T QCD Gross, Pisarski, Yaffe (1981)

aPBC: $m^2 = \frac{1}{3} g^2 T^2 (N_c + \frac{1}{2} N_F)$ ⇒ PBC: $m^2 = \frac{1}{3} g^2 T^2 (N_c - N_F)$

$N_F > N_c$ means tachyonic, leading to $\langle A_y \rangle \neq 0$ $\boxed{\text{GSB could be !}}$

- Hosotani mechanism Hosotani (1983)

$q_i \neq q_j$ ⇨ spontaneous gauge symmetry breaking

- Determined dynamically, depends on matter.
- Spectrum $m_n^2 = \frac{1}{L^2}(n + q_i - q_j)^2$ → massive gauge boson

e.g.) $q_1=q_2=q_3\neq q_4=q_5$, $SU(5)\rightarrow SU(3)\times SU(2)\times U(1)$

- Gauge-Higgs unification Manton (1979) Hosotani (1983)

Nonzero q breaks gauge symmetry in (4+1)D

→ Higgs as fluctuation of A_y ($m_H \sim O(g/L)$)

cf.) $SO(5)\times U(1)$ RS model Agashe, Contino, Pomarol (2005)

$SO(5)\times U(1) \rightarrow SO(4)\times U(1) \rightarrow SU(2)\times U(1) \rightarrow U(1)$

Orbifold b.c. brane dynamics Hosotani mech.

Stable Higgs: m_H=130 GeV

Hosotani mechanism has been eventually studied
in a different context.....,
although they did not focus on it.

Myers, Ogilvie (07)(08)(09)
Cossu, D'Elia(09)
Nishimura, Ogilvie(10)

QCD via Large-N volume reduction ? Eguchi-Kawai (1982)

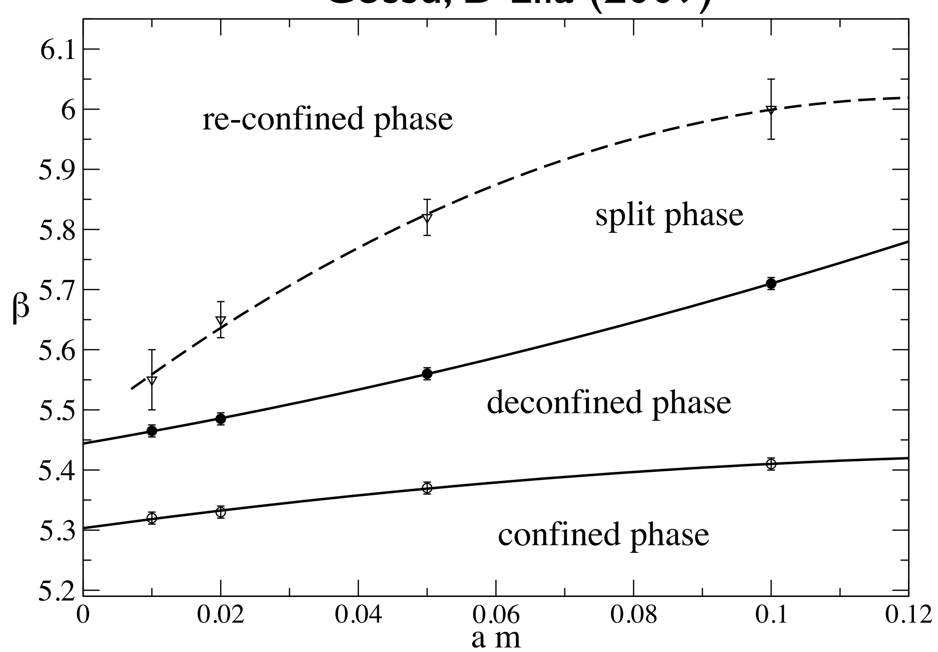
Z_3 confined phase survives at small L ? (volume-indep.)

- Pure Eguchi-Kawai : Z_3 broken Bhanot-Heller-Neuberger (1982)
- Eguchi-Kawai w/ adj. : Z_3 restored ? Kovtun-Unsal-Yaffe (2007)

QCD with PBC adj. matter is also a hot topic in the area.

- Lattice Finite-T QCD with PBC adj. matter Myers-Ogilvie (2007) Cossu-D'Elia (2009)

Cossu, D'Elia (2009)



- Rich phase structure found !
- Should be understood from Hosotani mechanism.

Purpose

1. Understand phase structure in gauge theory with PBC fermions, focusing on SGSB.

2. Obtain useful information for phenom. models.

3. Seek other setups leading to SGSB.

Tools
- One-loop effective potential
- Polyakov-loop-extended NJL

SU(N) perturbative one-loop effective potential

Gross-Pisarski-Yaffe (1981)

1. Replace $\tau \to y$, $\beta \to L$.
2. Wilson-loop phases $\to$ zero modes $\langle A_y \rangle = \frac{2\pi}{gL}\,\mathrm{diag}[q_1, \cdots, q_N]$

Gauge : $$\mathcal{V}_g(q) = -\frac{2}{L^4\pi^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{\cos[2n\pi q_{ij}]}{n^4}$$

Fund. : $$\mathcal{V}_f^{\phi}(q; N_f, m_f) = \frac{2N_f m_f^2}{\pi^2 L^2}\sum_{i=1}^{N}\sum_{n=1}^{\infty}\frac{K_2(n m_f L)}{n^2}\cos[2\pi n(q_i+\phi)]$$

Adj. : $$\mathcal{V}_a^{\phi}(q; N_a, m_a) = \frac{2N_a m_a^2}{\pi^2 L^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{K_2(n m_a L)}{n^2}\cos[2\pi n(q_{ij}+\phi)]$$

with $q_1+q_2+\cdots+q_{N-1}+q_N = 0 \quad (mod\ 1)$

SU(N) perturbative one-loop effective potential

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$q_i - q_j$

# of color

compacted scale

Fund. : $$\mathcal{V}_f^{\phi}(q; N_f, m_f) = \frac{2N_f m_f^2}{\pi^2 L^2}\sum_{i=1}^{N}\sum_{n=1}^{\infty}\frac{K_2(nm_f L)}{n^2}\cos[2\pi n(q_i+\phi)]$$

# of fund. flavor

Fund. mass

boundary condition

φ=0 : periodic

φ=1/2 : anti-periodic

Adj. : $$\mathcal{V}_a^{\phi}(q; N_a, m_a) = \frac{2N_a m_a^2}{\pi^2 L^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{K_2(nm_a L)}{n^2}\cos[2\pi n(q_{ij}+\phi)]$$

# of adj. flavor

Adj. mass

with $$q_1 + q_2 + \cdots + q_{N-1} + q_N = 0 \quad (mod\ 1)$$

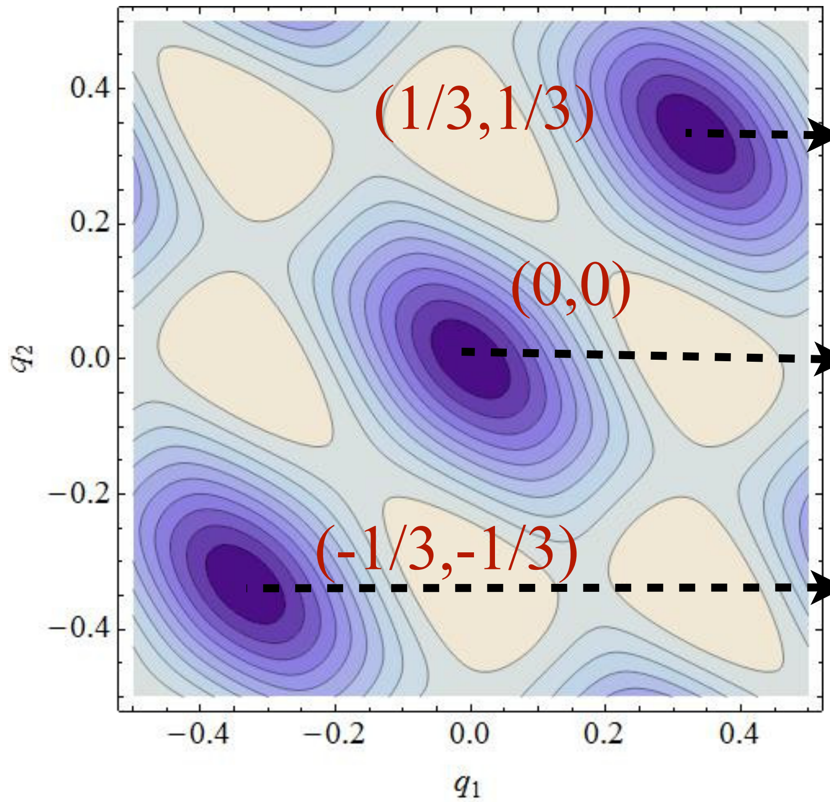
How to observe GSB

Look for global minima in effective potential

$q_1 + q_2 + q_3 = 0 \pmod 1$ → a function of q_1, q_2 as $\mathcal{V}(q_1, q_2)$

Ex.) Contour plot for SU(3) pure gauge on $R^3 \times S^1$

$\mathcal{V}(q_1, q_2) = \mathcal{V}_g$

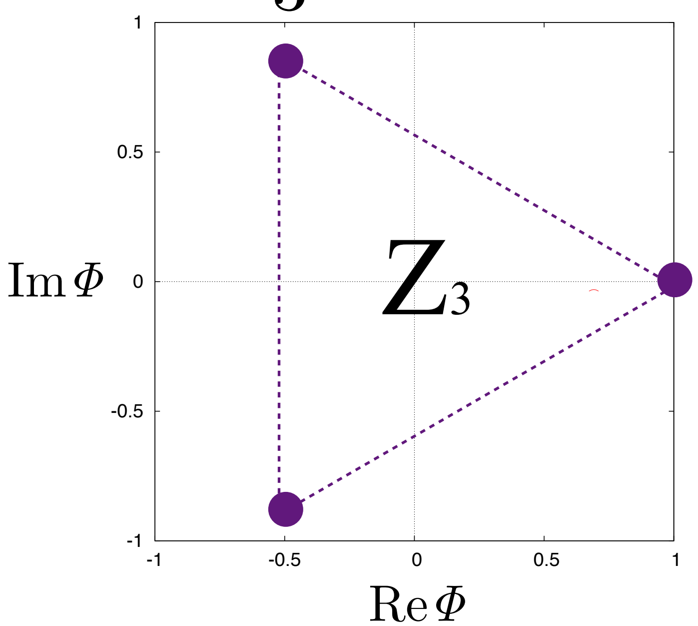


$(q_1, q_2, q_3) = (1/3, 1/3, 1/3)$

$(q_1, q_2, q_3) = (0, 0, 0)$

$(q_1, q_2, q_3) = (-1/3, -1/3, -1/3)$

$\Phi = \frac{1}{3}\mathrm{Tr}\,\exp[2\pi i q_i]$



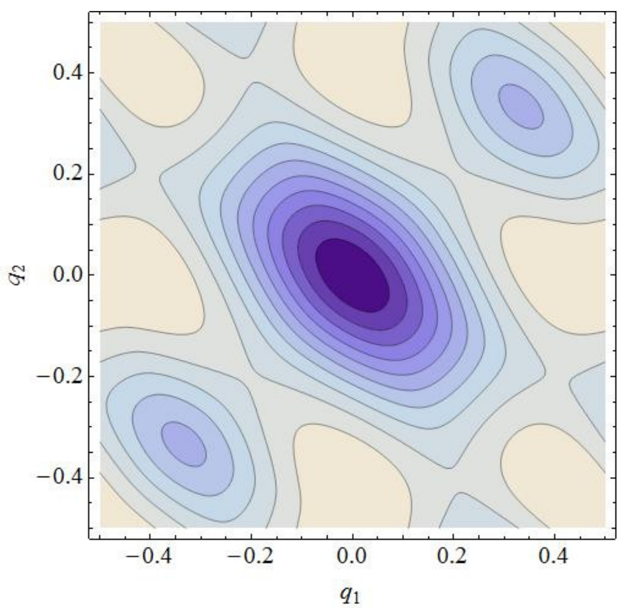
$\boxed{q_1 = q_2 = q_3}$ SU(3)

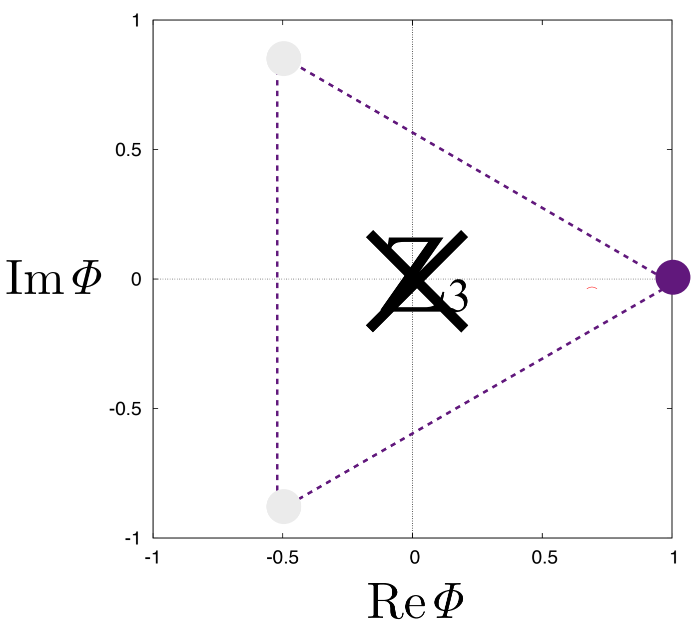
Polyakov-loop

Minima other than these three indicates GSB !

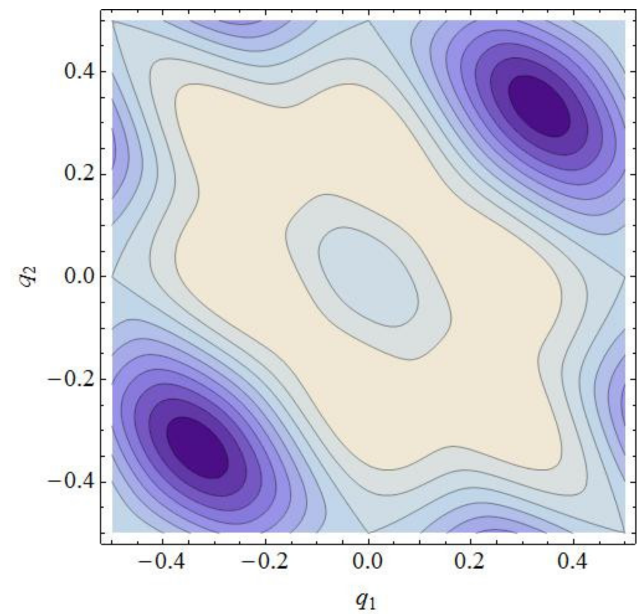
Non-GSB cases

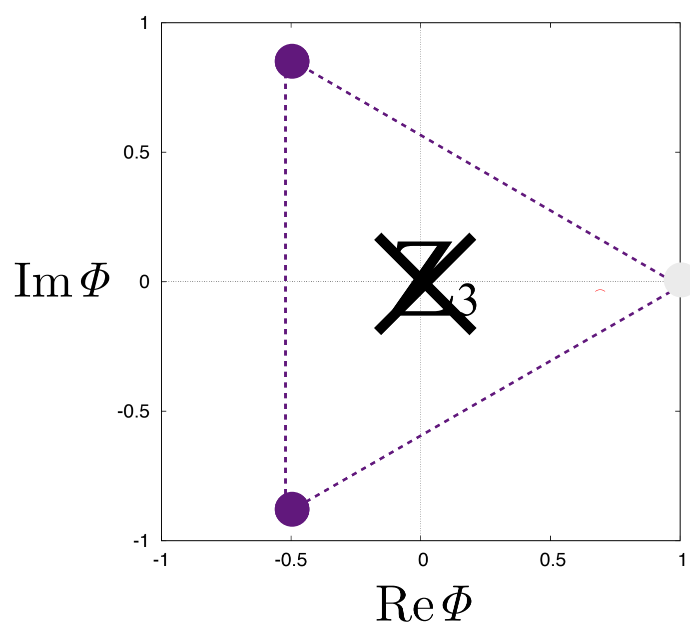
(1) aPBC fund.



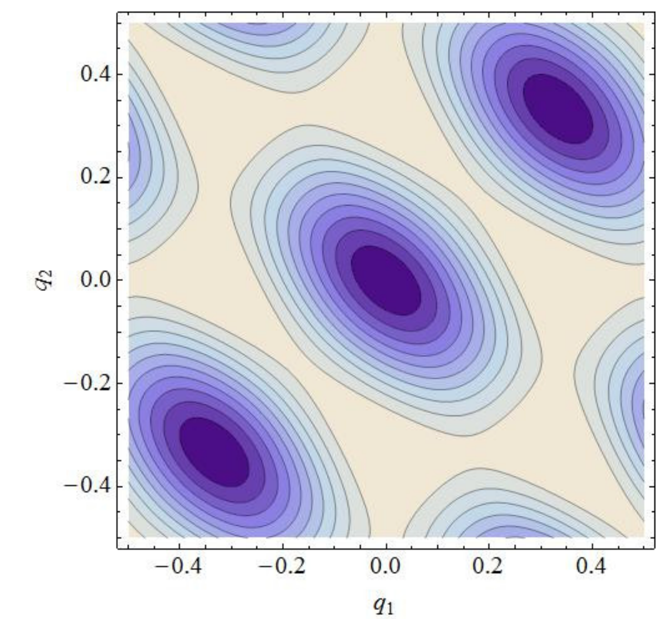


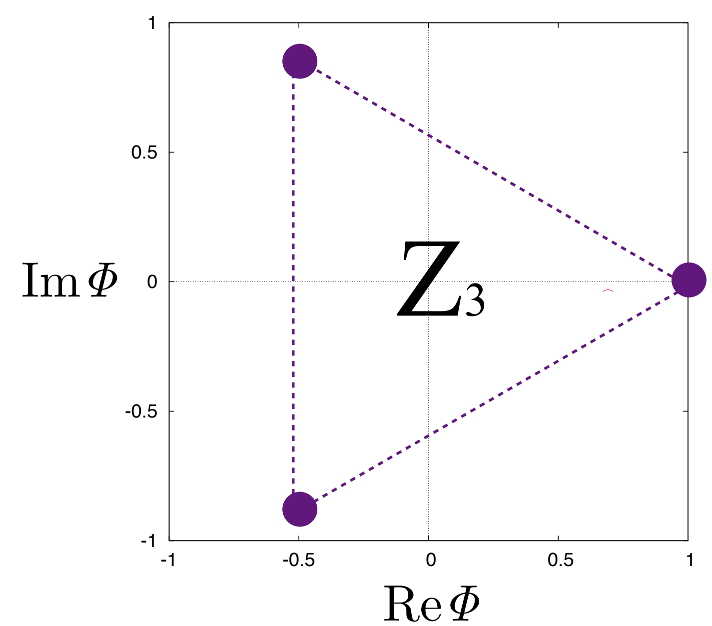
(2) PBC fund.





(3) aPBC adjoint





Gauge symmetry is unbroken.

4D Gauge Symmetry Breaking

(on $R^3 \times S^1$)

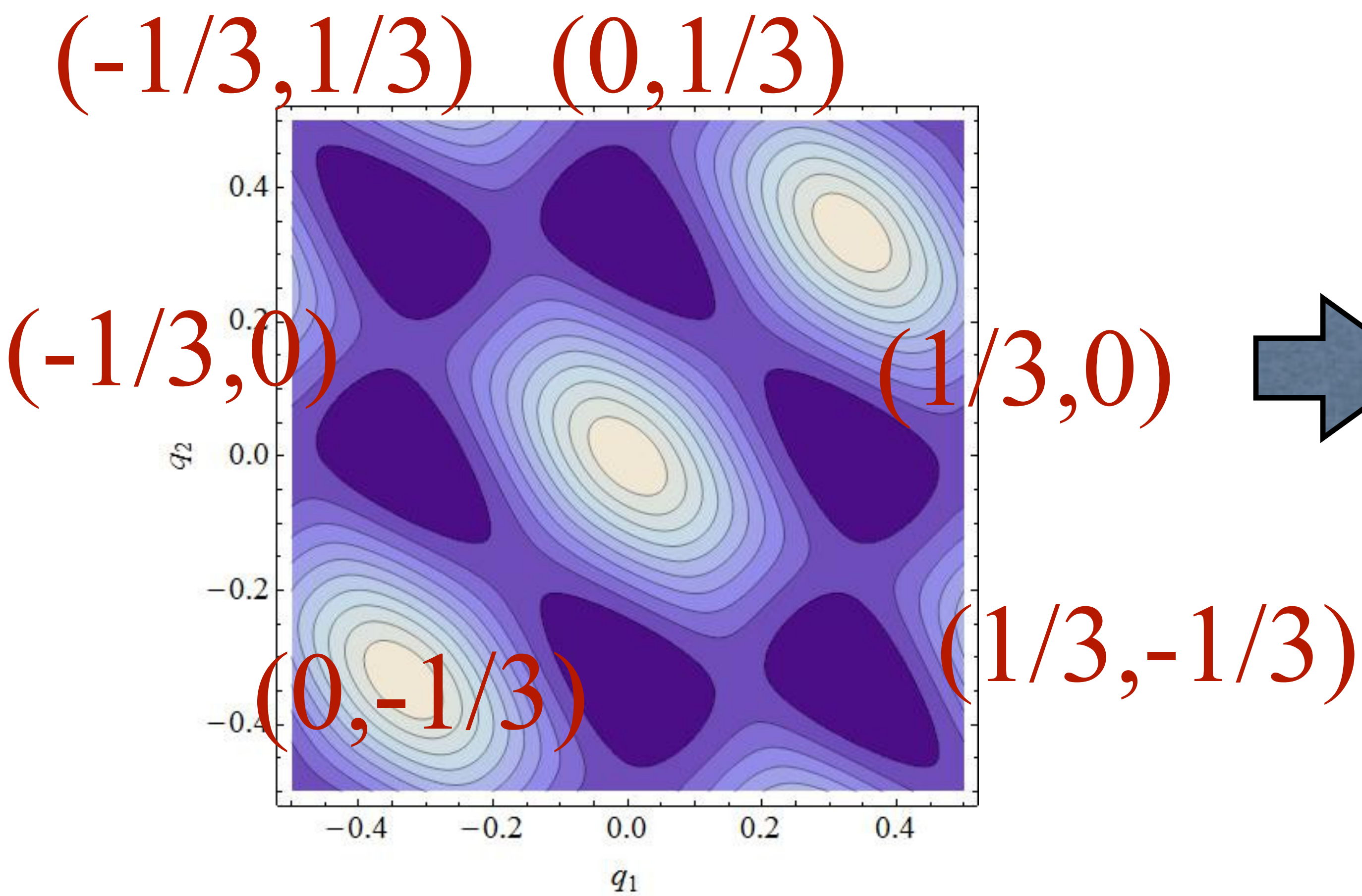
Kashiwa,TM [arXiv:1302.2196]

Cossu, Hatanaka, Hosotani, Itou, Noaki, work in progress

SU(3) with PBC adjoint (m=0)

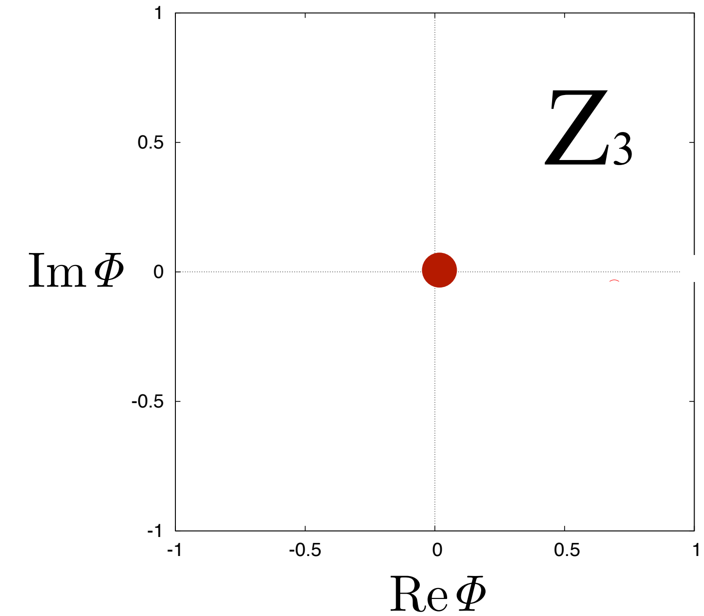
- Center Z_3 is unbroken.
- Polyakov-loop reflects Z_3

$$\mathcal{V}(q_1, q_2) = \mathcal{V}_g + \mathcal{V}_a^0$$



$$(q_1, q_2, q_3) = (1/3, -1/3, 0)$$

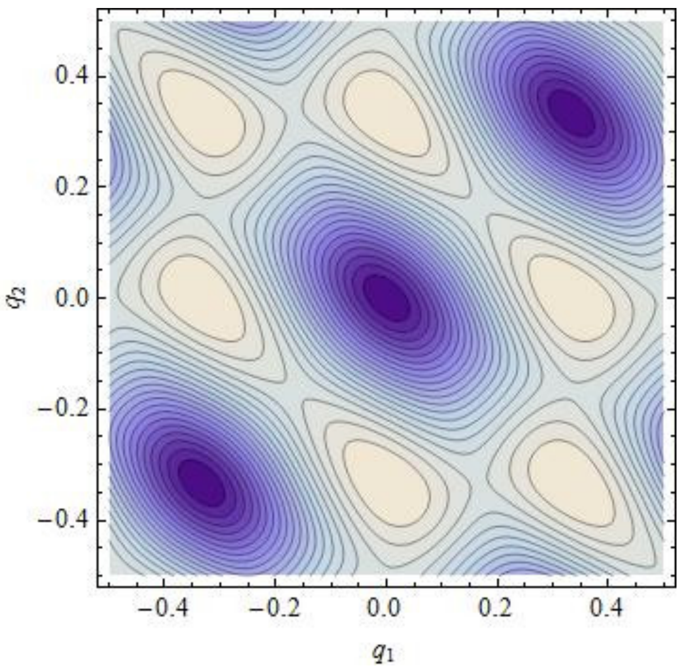
$$\boxed{q_1 \neq q_2 \neq q_3}$$



SU(3) gauge symmetry broken to U(1)×U(1)

Hosotani (1983)

SU(3) with PBC adjoint (m≠0)

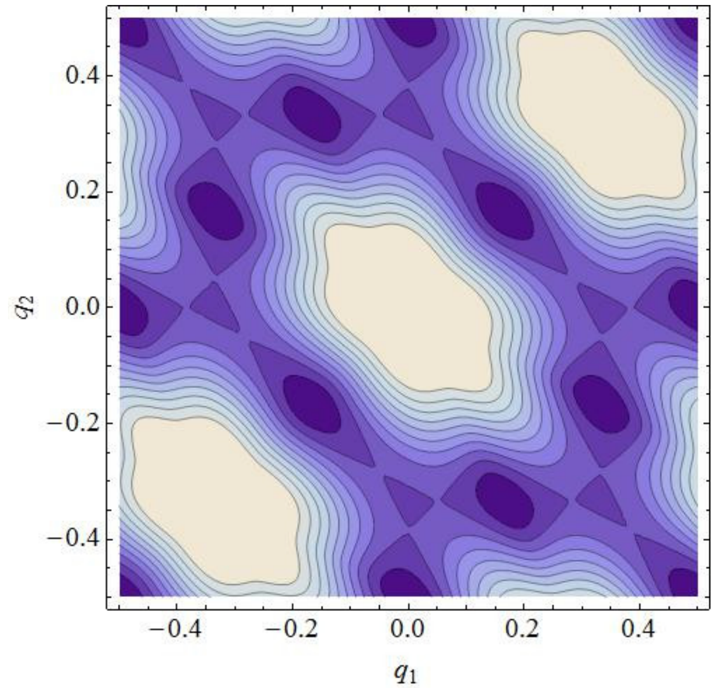


large m

$$(\frac{1}{3},\frac{1}{3},\frac{1}{3})\ (0,0,0)\ (-\frac{1}{3},-\frac{1}{3},-\frac{1}{3})$$

$$q_1 = q_2 = q_3$$

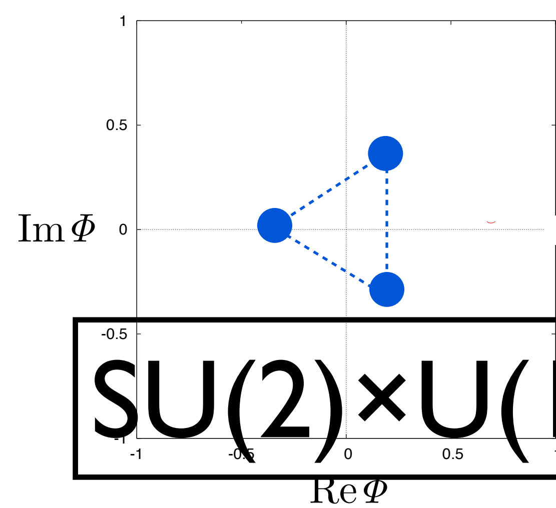
SU(3)



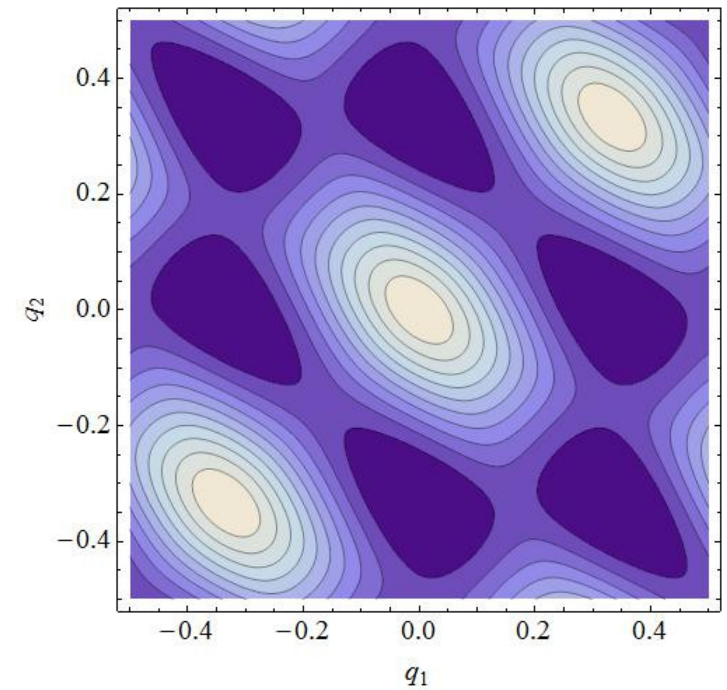
medium m

$$(\frac{1}{2},\frac{1}{2},0)\ (\frac{1}{6},\frac{1}{6},-\frac{1}{3})\ (-\frac{1}{6},-\frac{1}{6},\frac{1}{3})$$

$$q_1 = q_2 \neq q_3$$



SU(2)×U(1)



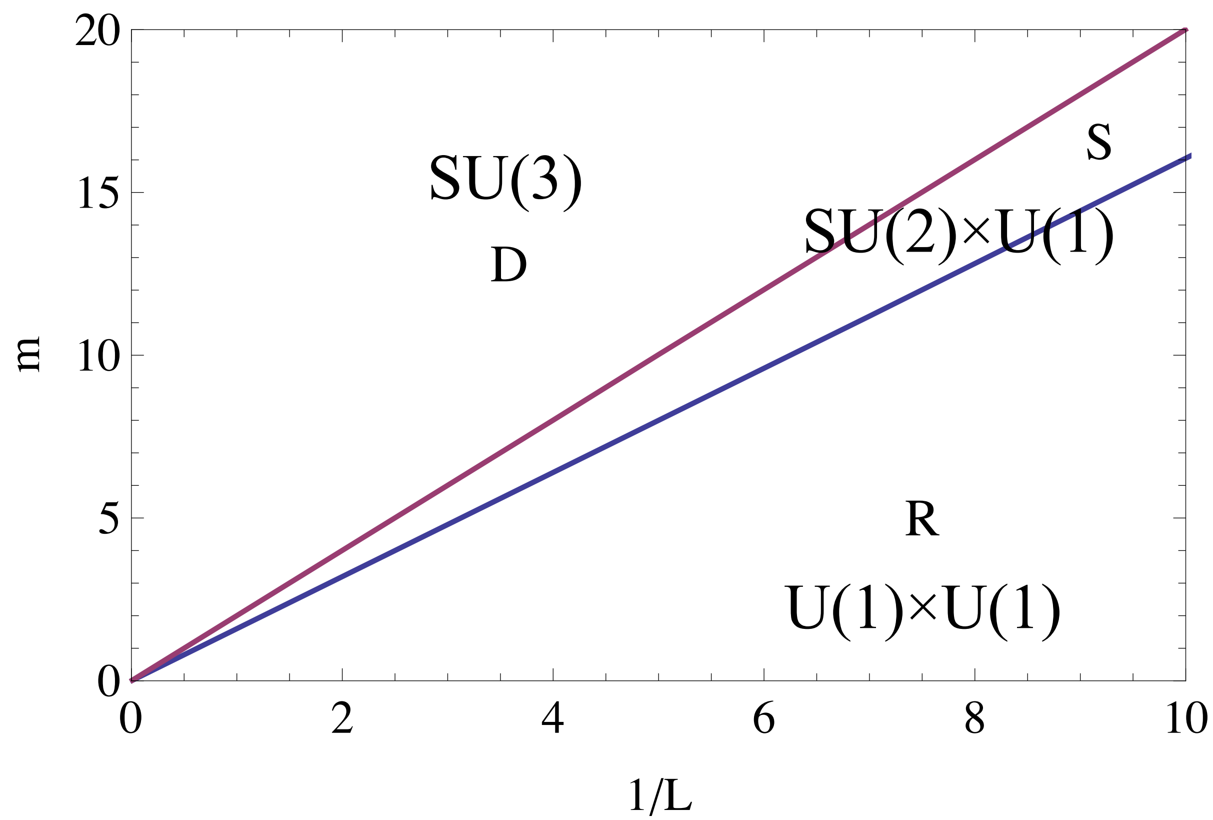
small m

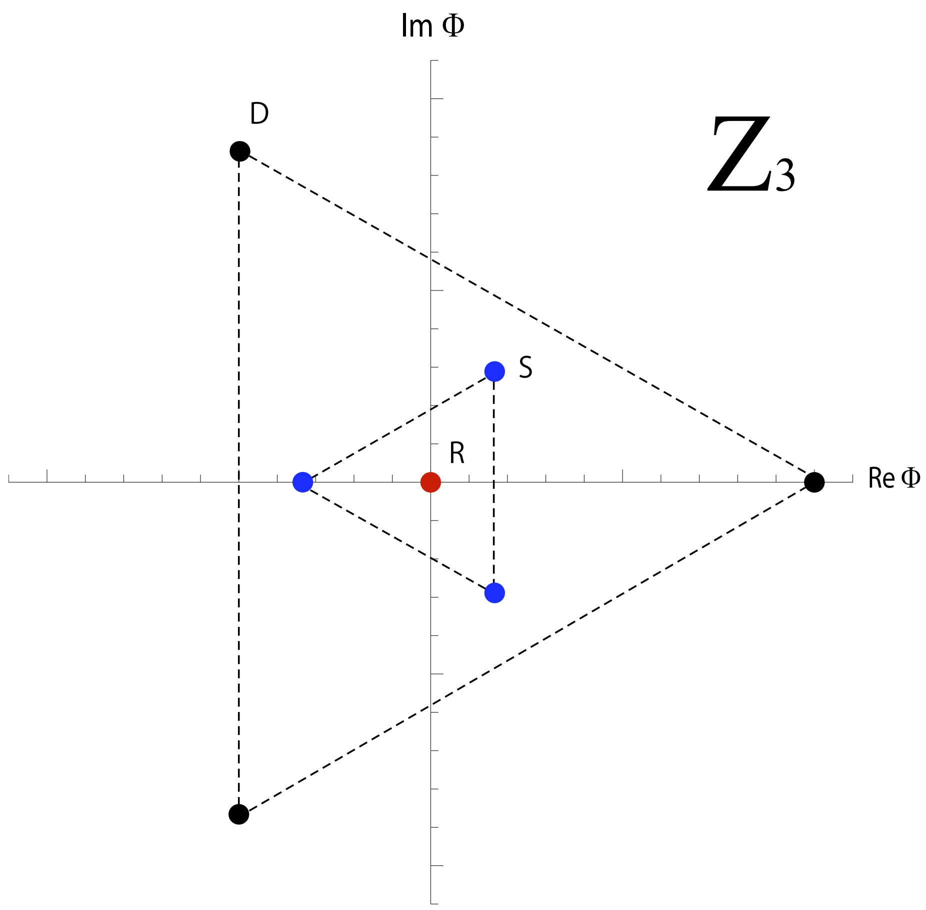
$$(\frac{1}{3},-\frac{1}{3},0)$$

$$q_1 \neq q_2 \neq q_3$$

U(1)×U(1)

Phase diagram & Polyakov-loop





SU(3) → "Deconfined"

SU(2)×U(1) → "Split" (Phenomenologically most desirable)

U(1)×U(1) → "Re-confined" (zero Polyakov-loop $\Phi=0$)

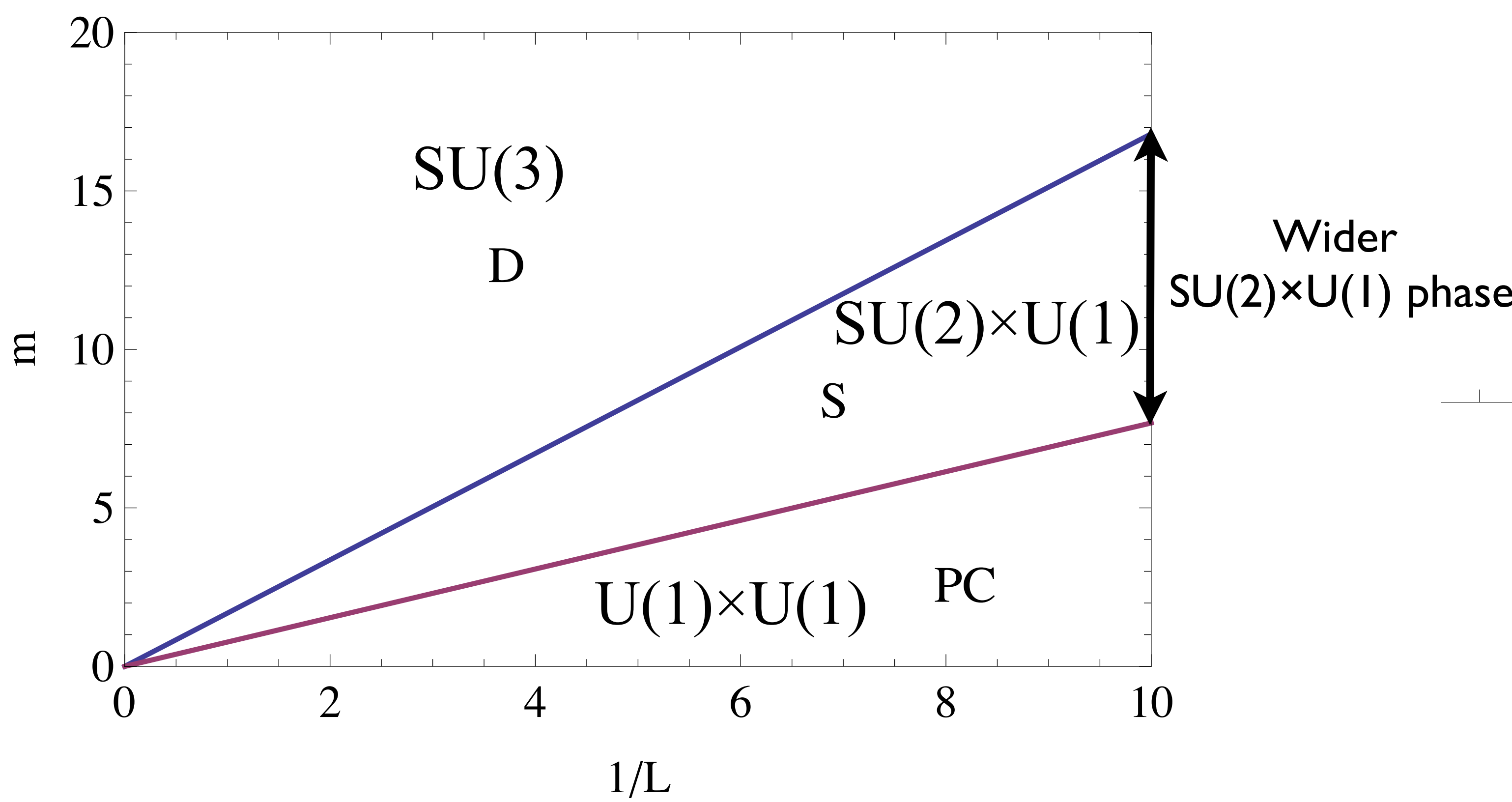
cf.) $\langle\Phi\rangle=0$ in confined phase is due to large fluctuation in strong-coupling regime.

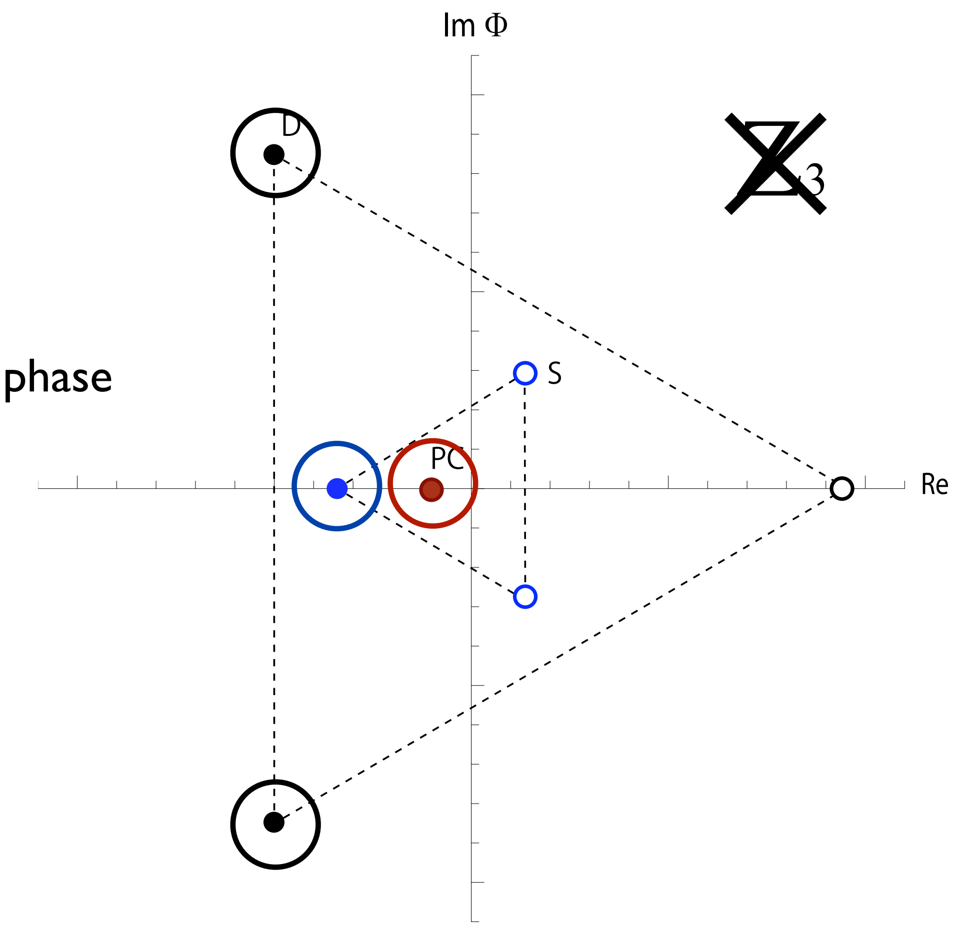
SU(2)×U(1) phase can be enlarged? →yes! By fund. matters

SU(3) with adj. & fund. with PBC

- *The center is broken by fund. matter.*
- *Vacua moved to* ReΦ<0 *direction.*

$$\mathcal{V}(q_1, q_2) = \mathcal{V}_g + \mathcal{V}_a^0 + \mathcal{V}_f^0$$

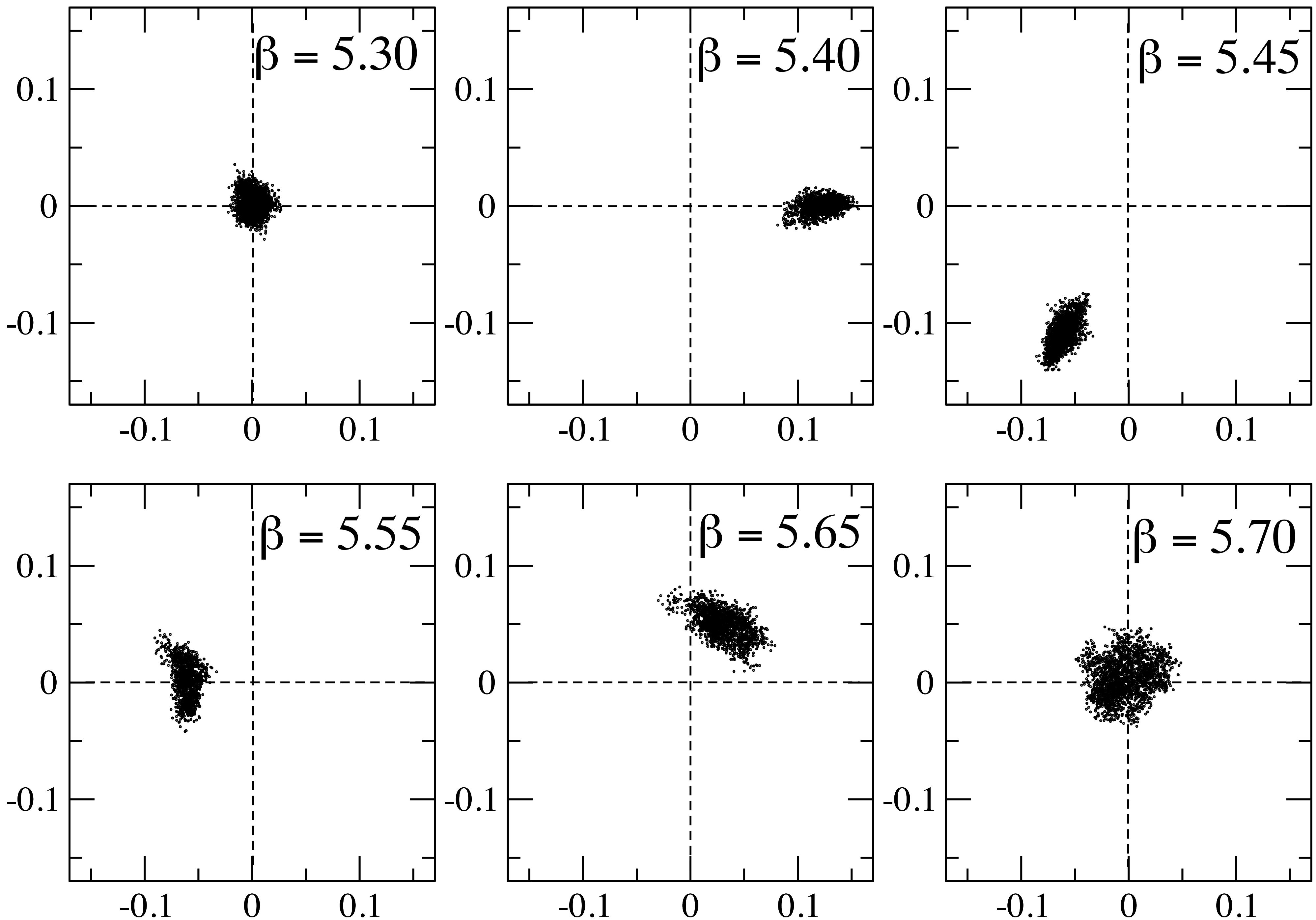




- SU(2)×U(1) minimum: deeper and stable due to Z_3 breaking
- U(1)×U(1) minimum: moves and unstable (approaching to S)

Re-interpretation of adj. lattice results

Cossu, D'Elia (2009)



Re-interpretation of adj. lattice results

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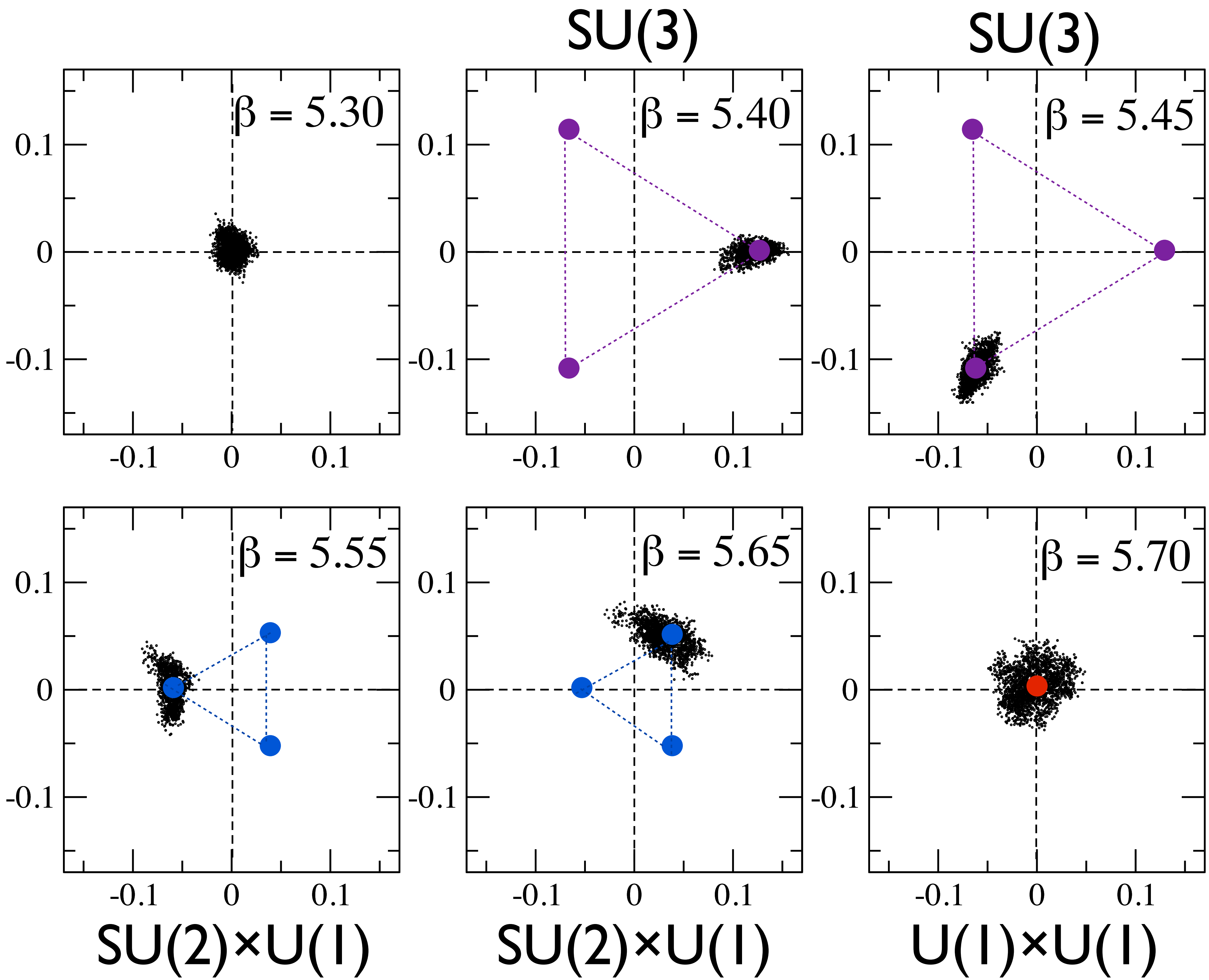


Chiral properties

- Chiral model with PBC adjoint 2 flavors Nishimura, Ogilvie (2010)

$$\mathcal{L}_{\rm PNJL} = \bar{\psi}(\gamma_\mu D_\mu + m)\psi - g_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] + \mathcal{V}_g \quad (D_\mu = \partial_\mu + iA_4)$$

Fukushima (2004)

$$\Rightarrow \mathcal{V}(q_1, q_2, \sigma) = \mathcal{V}_g + \mathcal{V}_F + \mathcal{V}_\chi$$

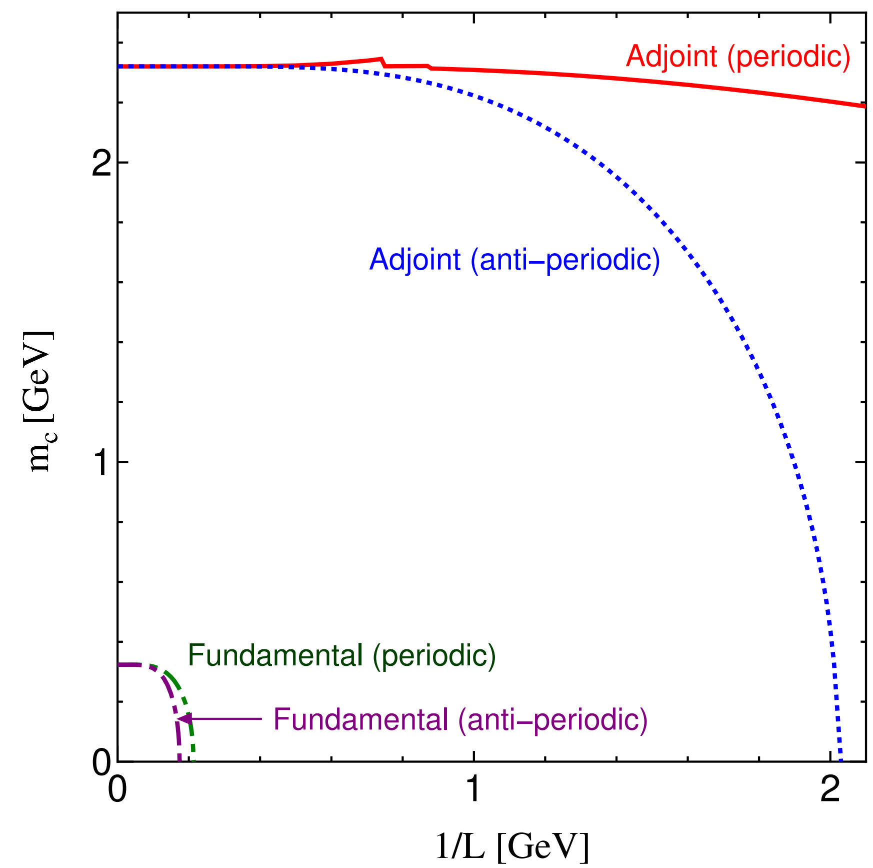
- Dimension parameters $\Lambda_{\rm cutoff}, \ g_S$
- fixed so as to reproduce aPBC results

Karsch, Lutgemeier (1998)

Fund: $\Lambda = 0.63$ GeV and $g_S\Lambda^2 = 2.19$

Adj: $\Lambda = 23.22$ GeV and $g_S\Lambda^2 = 0.63$

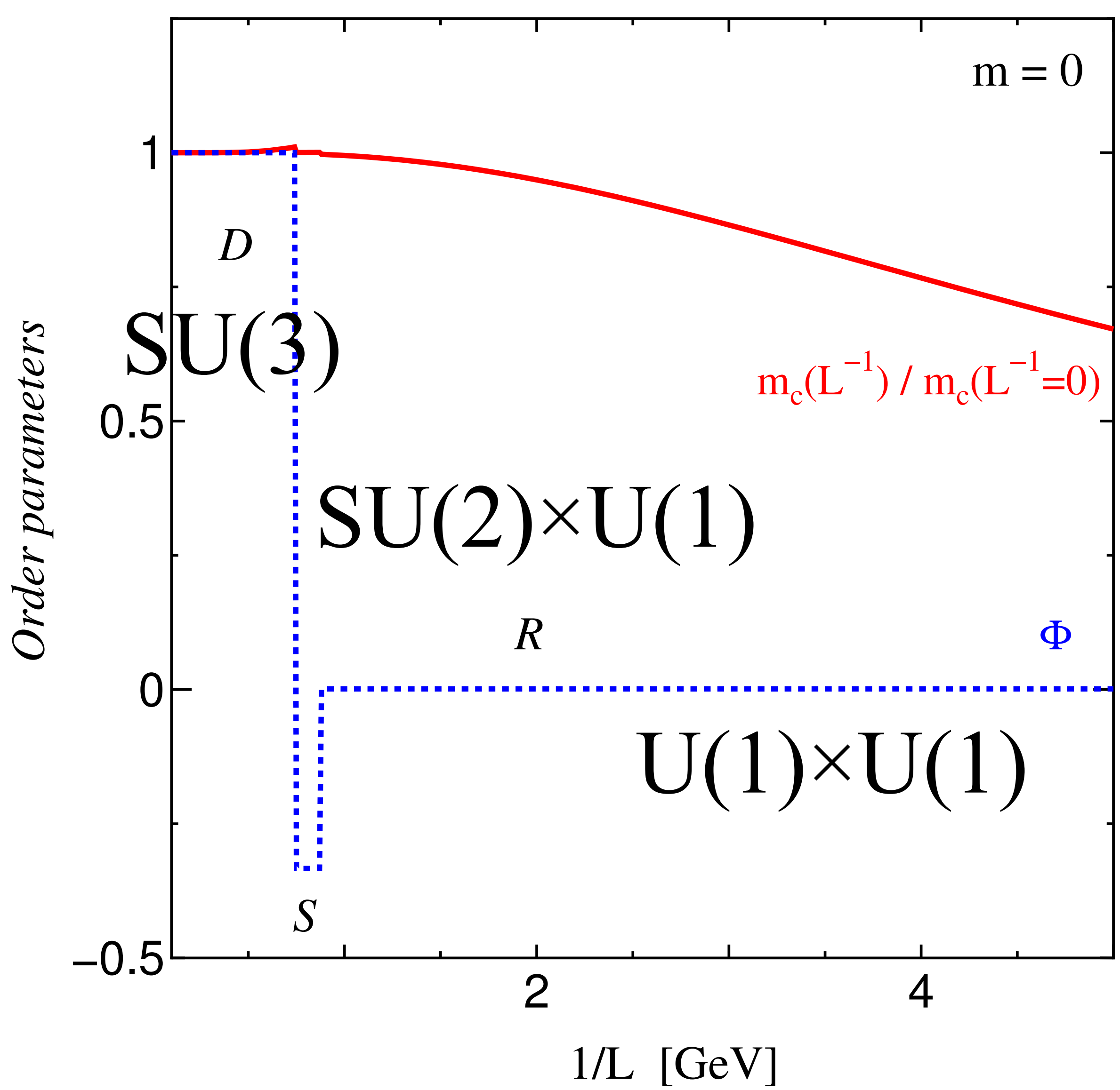
PBC adj. vs others



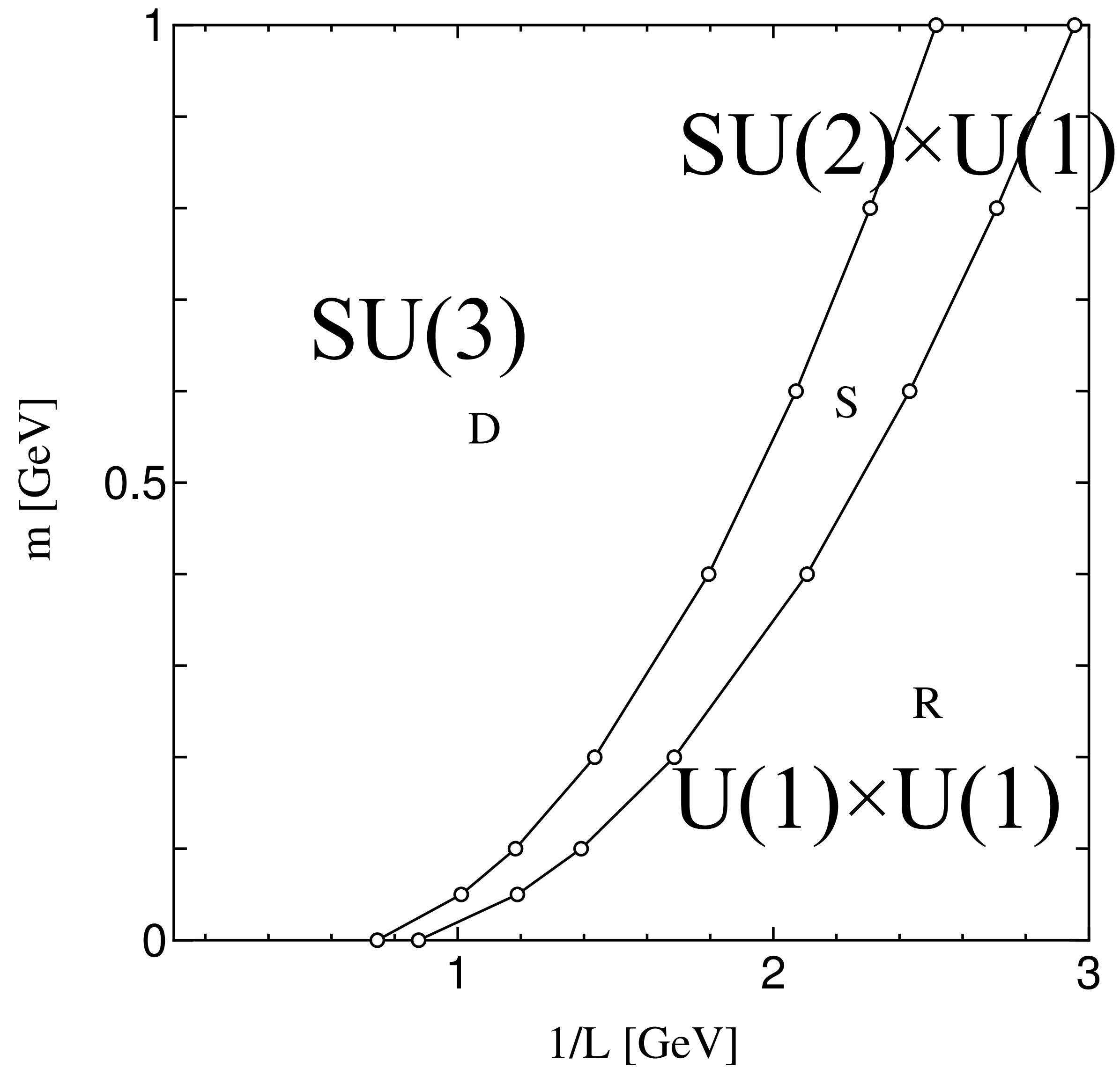
Chiral symmetry restored slowly for PBC adj. Cossu, D'Elia (2009)

Phase diagram w/ chiral part

chiral vs Re Φ



Phase diagram



- Qualitatively unchanged, but the scaling disappears.

5D Gauge Symmetry Breaking

(on $R^4 \times S^1$)

Kashiwa,TM [arXiv:1302.2196]

Cossu, Hatanaka, Hosotani, Itou, Noaki, work in progress

5D SU(N) one-loop effective potential

1. Replace $\tau \to y,\ \beta \to L.$
2. Wilson-loop phases → zero modes $\langle A_y \rangle = \dfrac{2\pi}{gL}\,\mathrm{diag}[q_1, \cdots, q_N]$

Gauge : $\mathcal{V}_g = -\dfrac{9}{4\pi^2 L^5} \sum_{i,j=1}^{N} \sum_{n=1}^{\infty} \left(1 - \dfrac{1}{N}\delta_{ij}\right) \dfrac{\cos(2\pi n q_{ij})}{n^5}$

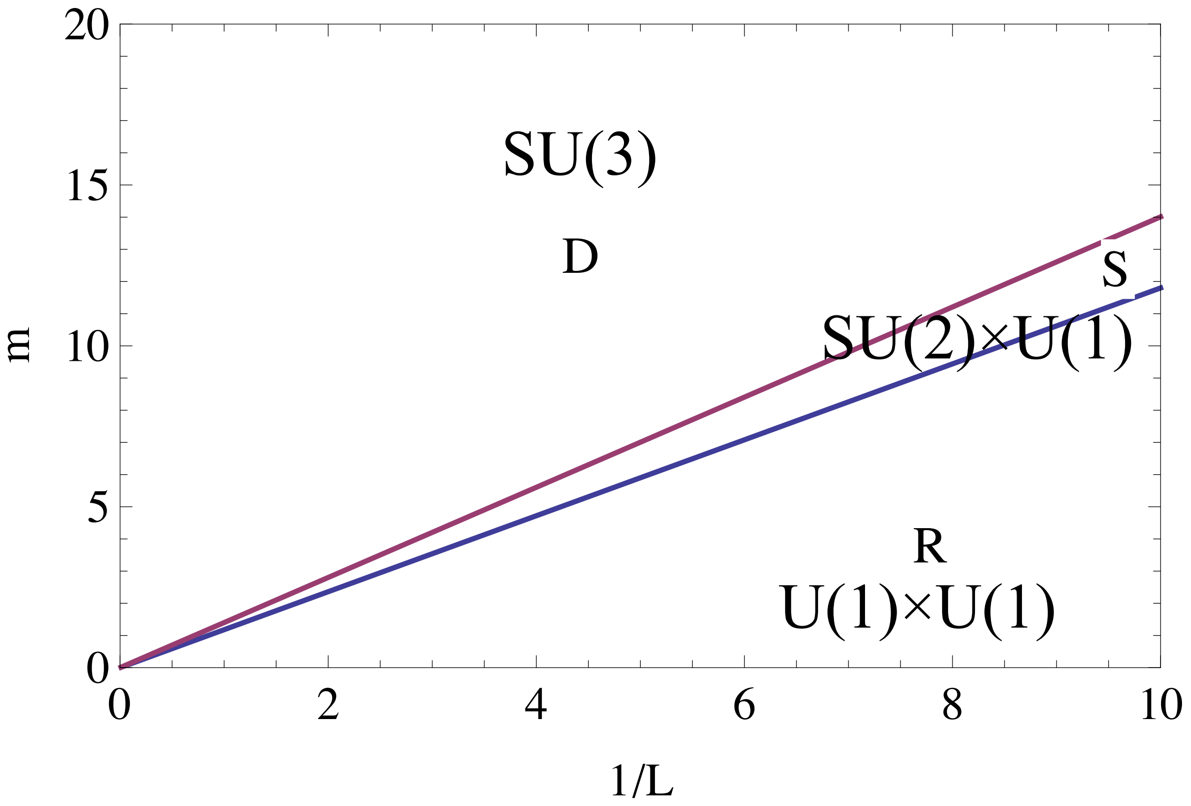
Fund. : $\mathcal{V}_f^{\phi}(N_f, m_f) = \dfrac{\sqrt{2} N_f (m_f/L)^{5/2}}{\pi^{5/2}} \sum_{i=1}^{N} \sum_{n=1}^{\infty} \dfrac{K_{5/2}(n m_f L)}{n^{5/2}} \cos[2\pi n (q_i + \phi)]$

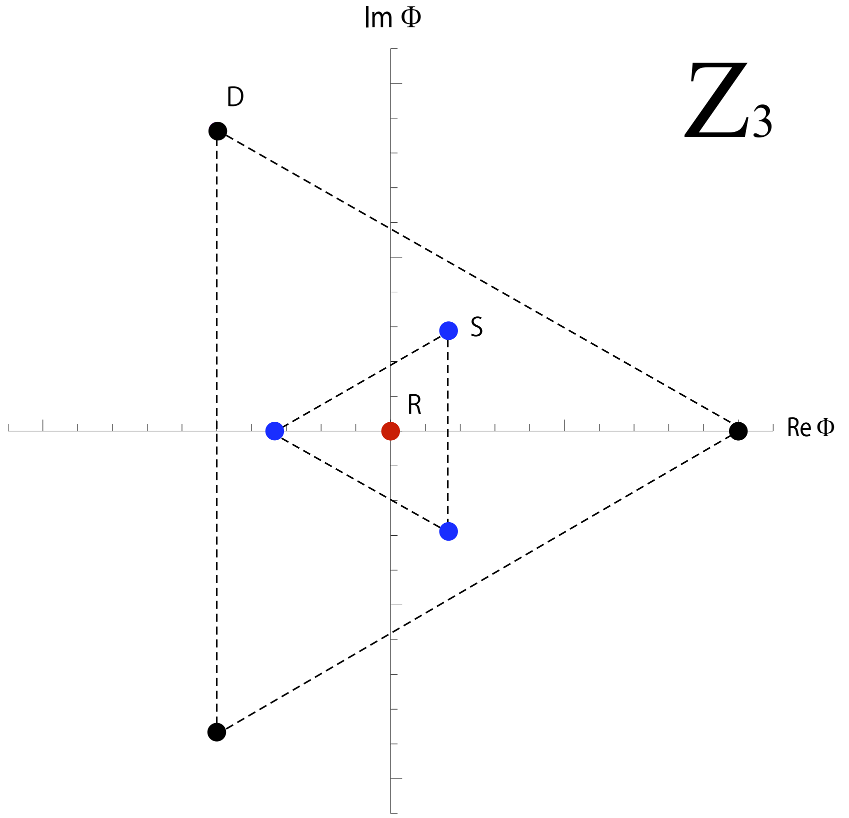
Adj. : $\mathcal{V}_a^{\phi}(N_a, m_a) = \dfrac{\sqrt{2} N_a (m_a/L)^{5/2}}{\pi^{5/2}} \sum_{i,j=1}^{N} \sum_{n=1}^{\infty} \left(1 - \dfrac{1}{N}\delta_{ij}\right) \dfrac{K_{5/2}(n m_a L)}{n^{5/2}} \cos[2\pi n (q_{ij} + \phi)]$

with $q_1 + q_2 + \cdots + q_{N-1} + q_N = 0 \pmod 1$

5D SU(3) with adjoint PBC

$$\mathcal{V}(q_1, q_2) = \mathcal{V}_g + \mathcal{V}_a^0$$



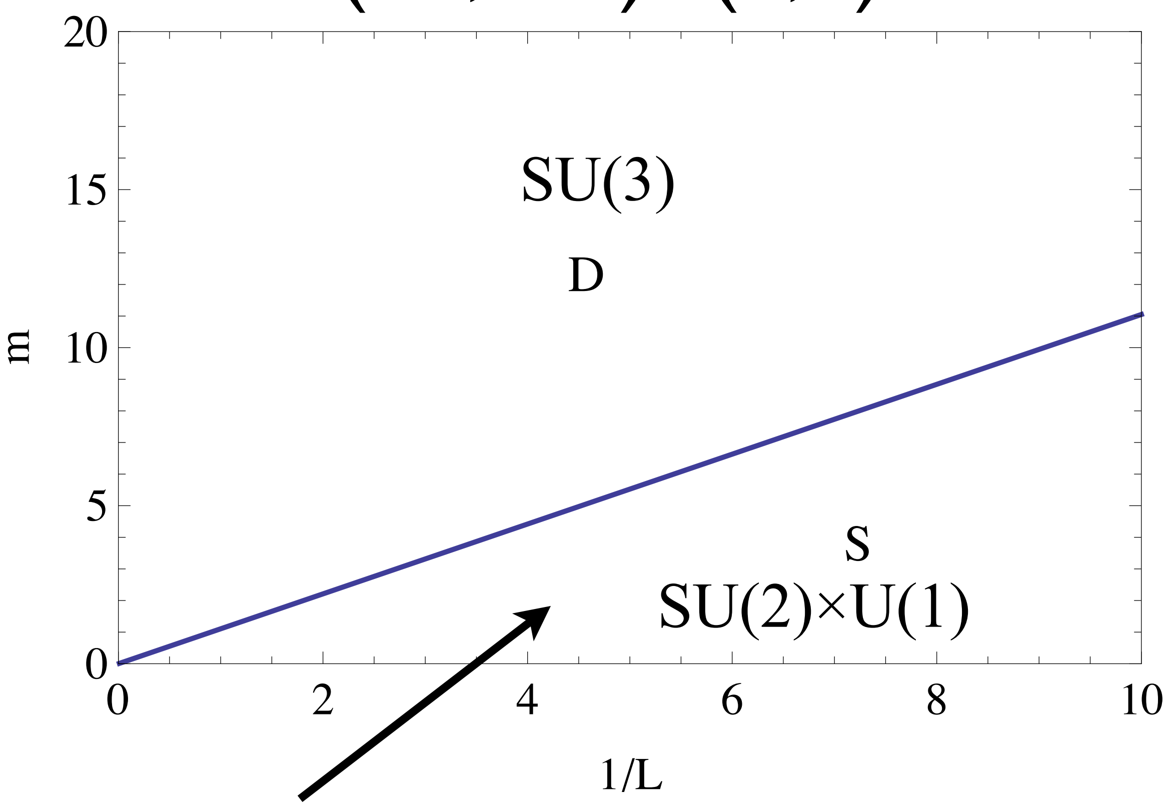


- Qualitatively the same as 4D, but narrower SU(2)×U(1).
- Z_3 symmetric Polyakov-loop distribution is the same.

5D SU(3) w/ adj. & fund.

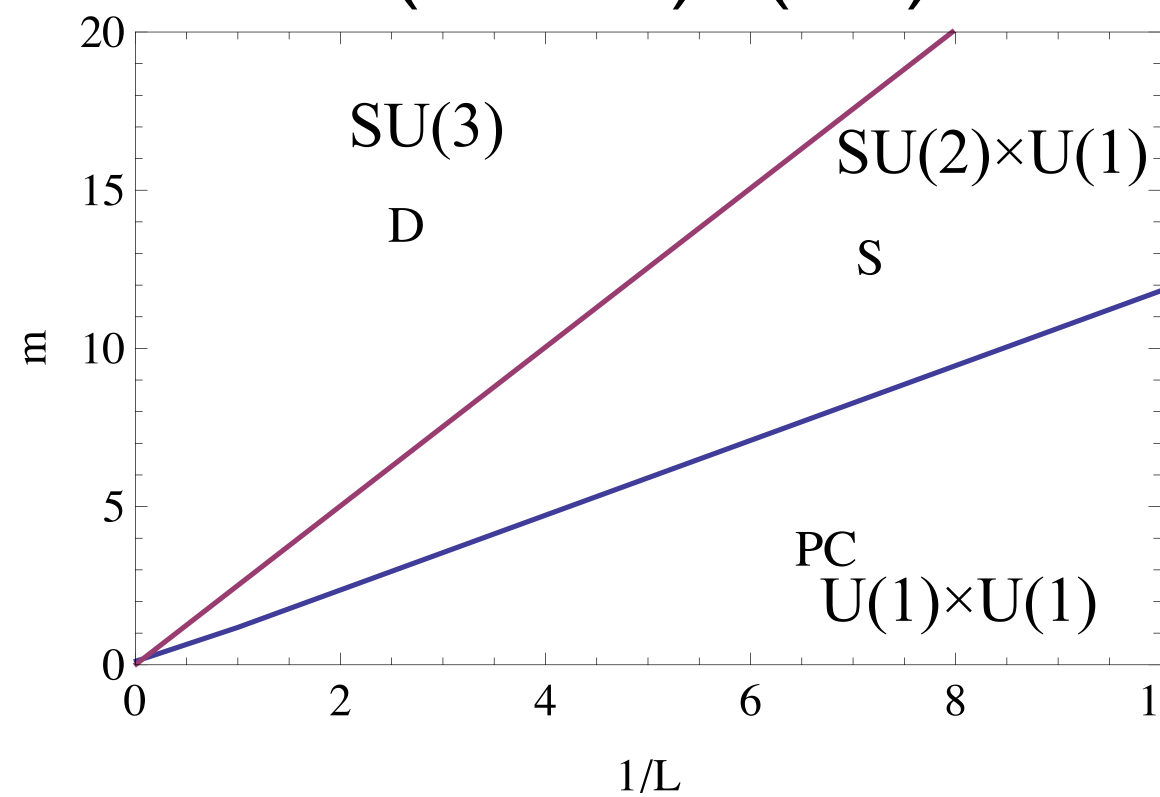
$$\mathcal{V}(q_1, q_2) = \mathcal{V}_g + \mathcal{V}_a^0 + \mathcal{V}_f^0$$

(Nf, Na)=(1,1)



Split phase dominant !

(Nf, Na)=(2,2)



- SU(2)×U(1) phase enhancement is more prominent.
- Phase structure is more sensitive to # of flavors.

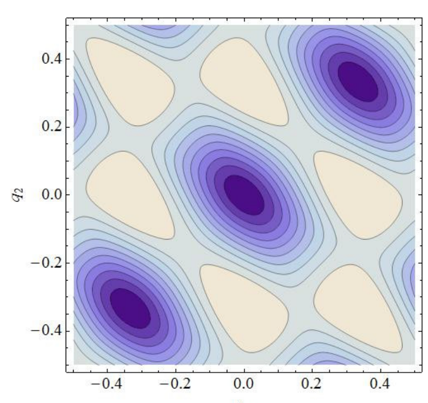
What happens in the potential ?

Competition of adj. and gluon effective potentials

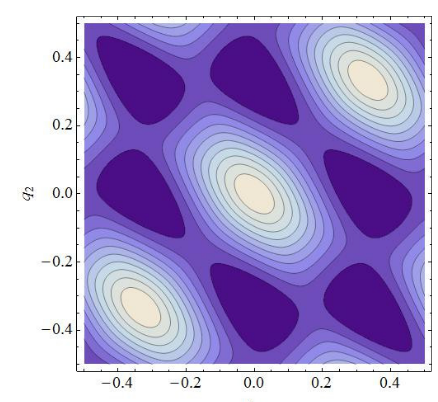
Gluon $$\mathcal{V}_g = - \frac{2}{L^4\pi^2} \sum_{i,j=1}^{3} \sum_{n=1}^{\infty} \left(1 - \frac{1}{3}\delta_{ij}\right) \frac{\cos[2n\pi q_{ij}]}{n^4}$$

opposite sign !

PBC adj. $$\mathcal{V}_a = + \frac{4}{L^4\pi^2} \sum_{i,j=1}^{3} \sum_{n=1}^{\infty} \left(1 - \frac{1}{3}\delta_{ij}\right) \frac{\cos[2\pi n q_{ij}]}{n^4}$$



$\mathcal{V}_g$



$\mathcal{V}_g + \mathcal{V}_a^0$

$\boxed{Z_3 \text{ unbroken}}$

Can be realized by Fund. quarks with appropriate B.C. ?

The answer is Yes !

Flavor-dependent twisted B.C. (FTBC)

Kouno, TM, Kashiwa, Makiyama, Sasaki, Yahiro, in preparation.

FTBC for 3 fundamental flavors in SU(3)

Sakai, Kouno, Sasaki, Yahiro (2012)

$$\boxed{(q_1, q_2, q_3)_{x,y+L} = (q_1, e^{2\pi i/3} q_2, e^{4\pi i/3} q_3)_{x,y}}$$

cf.) Flavored chemical potential

$\downarrow$ Z_3 transformation

$$(e^{2\pi i/3} q_1, e^{4\pi i/3} q_2, q_3)_{x,y}$$

Relabeling

Z_3 center is preserved by use of Z_3 of flavor SU(3).

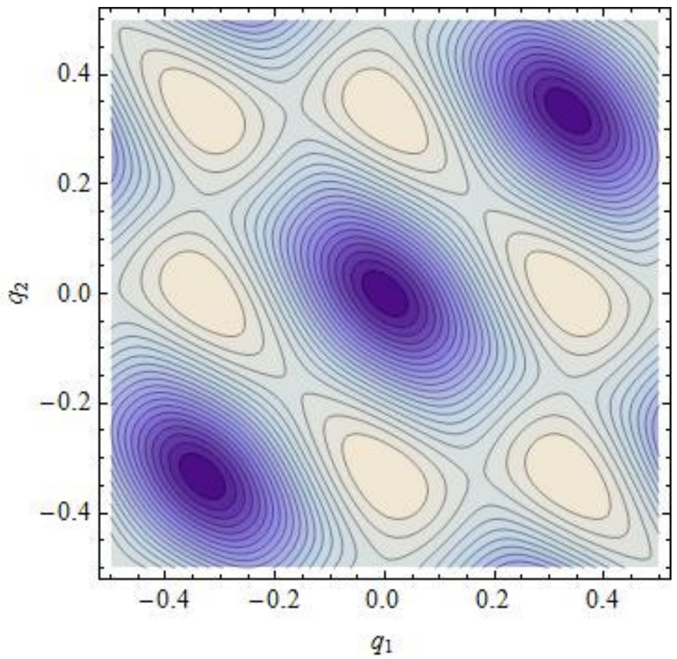
- One-loop effective potential for FTBC

$$\mathcal{V}_f^{FT} = +\frac{4}{L^4\pi^2} \sum_i^3 \sum_f^3 \sum_{n=1}^{\infty} \frac{\cos[2\pi n q_{if}]}{n^4} \qquad q_{if} = q_i + (f-1)/3$$

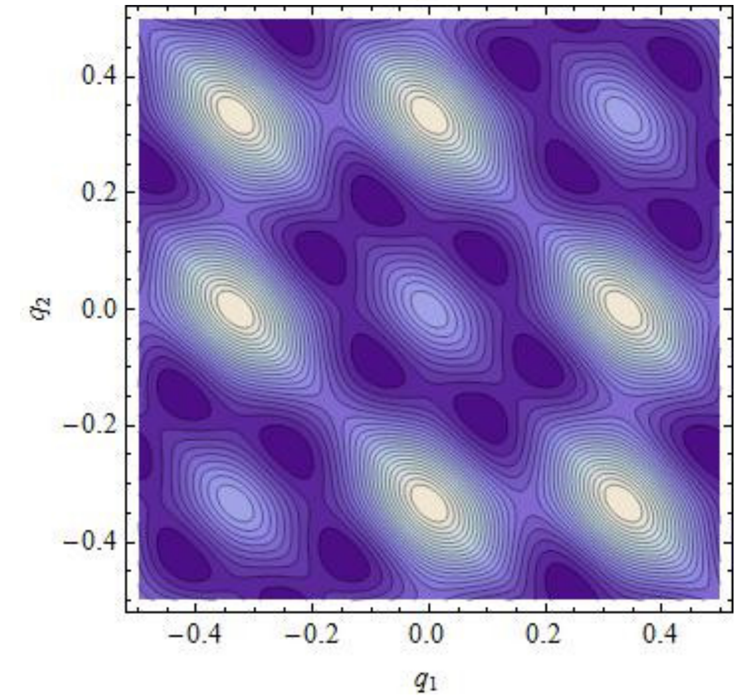
similar form to adj. case

cf.) $\mathcal{V}_a = +\frac{4}{L^4\pi^2} \sum_{i,j=1}^{3} \sum_{n=1}^{\infty} \left(1 - \frac{1}{3}\delta_{ij}\right) \frac{\cos[2\pi n q_{ij}]}{n^4}$

GSB and phase structure for FTBC



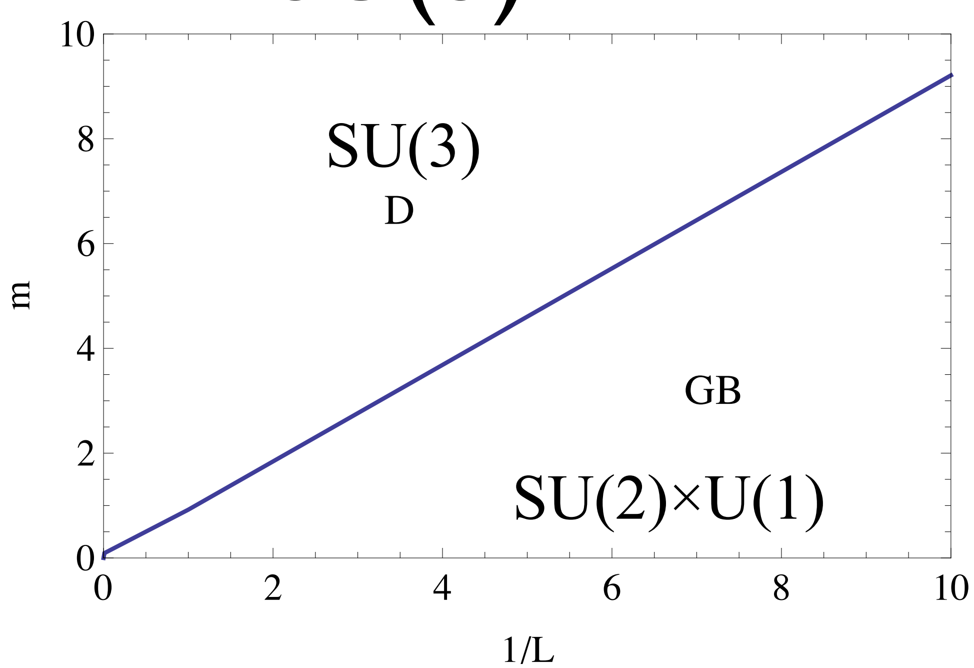
large m



small m

SU(3)

SU(2)×U(1)

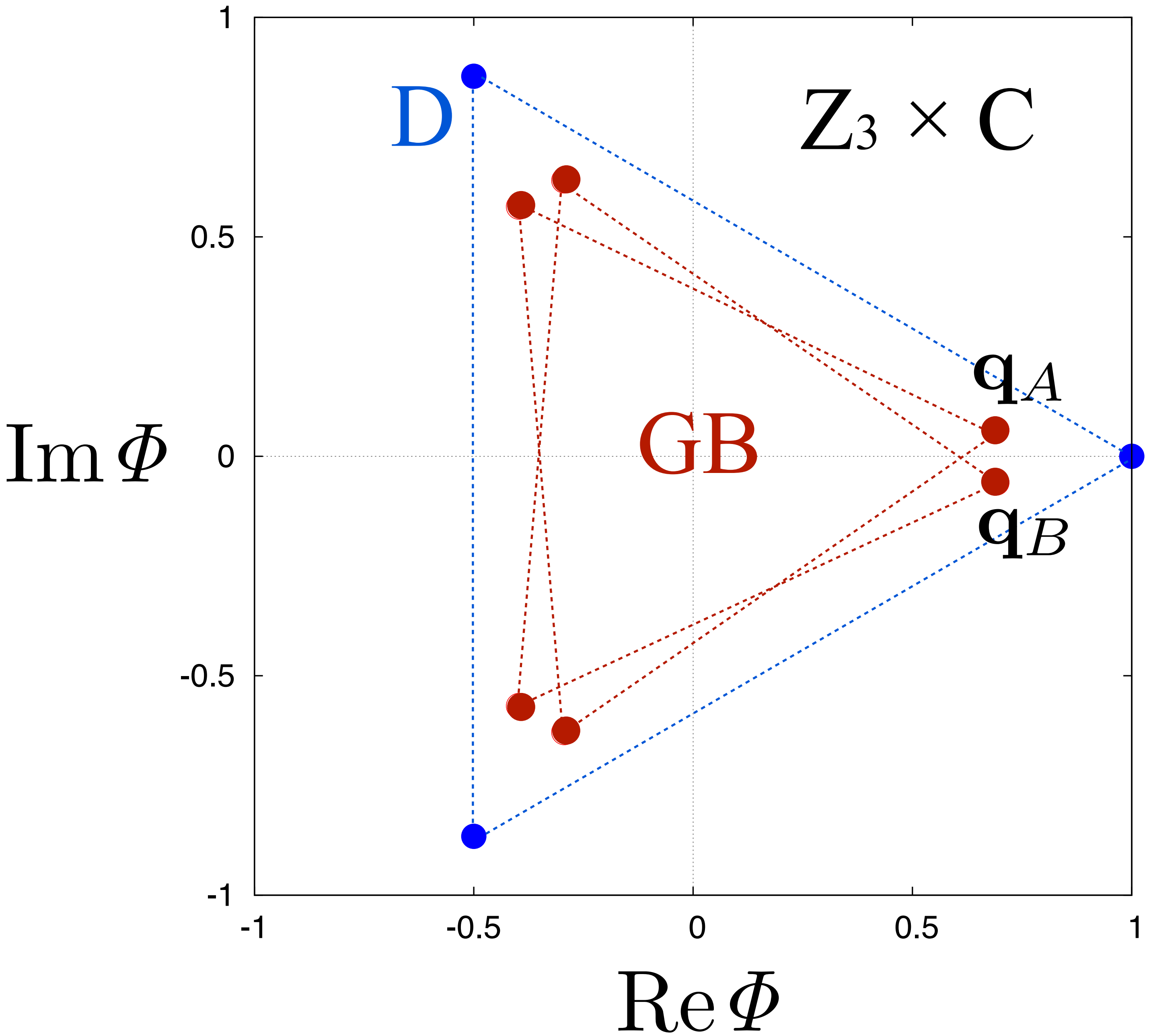


$$
\begin{aligned}
(q_1, q_2, q_3)_1 =&(\alpha/9, \alpha/9, -2\alpha/9),\\
&((\alpha+3)/9, (\alpha+3)/9, (3-2\alpha)/9),\\
&(-(3-\alpha)/9, -(3-\alpha)/9, (6-2\alpha)/9)\\
(q_1, q_2, q_3)_2 =& -(\alpha/9, \alpha/9, -2\alpha/9),\\
&-((\alpha+3)/9, (\alpha+3)/9, (3-2\alpha)/9),\\
&-(-(3-\alpha)/9, -(3-\alpha)/9, (6-2\alpha)/9)
\end{aligned}
$$

SU(3) broken only to SU(2)×U(1) !

$$q_1 = q_2 \neq q_3$$

Polyakov loop distribution



- Six possible vacua in GB phase
- They are paired as

$$(q_1, q_2, q_3)_A = -(q_1, q_2, q_3)_B$$

$$(A_\mu \to -A_\mu,\ \mathrm{Im}\Phi \to -\mathrm{Im}\Phi)$$

- Can be interpreted as C pairs.

FTBC is flavor-imaginary-chemical potential.

Charge conjugation is also spontaneously broken !

Summary

1. Rich phase structure with SGSB in gauge theory on compactified space with PBC.

2. Fundamental flavors with PBC works to enhance SU(2)×U(1) phase.

3. Fund. fermions with FTBC also leads to SU(2)×U(1) SGSB.

4. Specific chiral properties.

Fusion topic between In-medium QCD and BSM !

Future works

- Further lattice study 4D to check our results.
- Lattice study for 5D as cutoff theory.
- Application of FTBC to BSM, QCD....

Elitzur's theorem

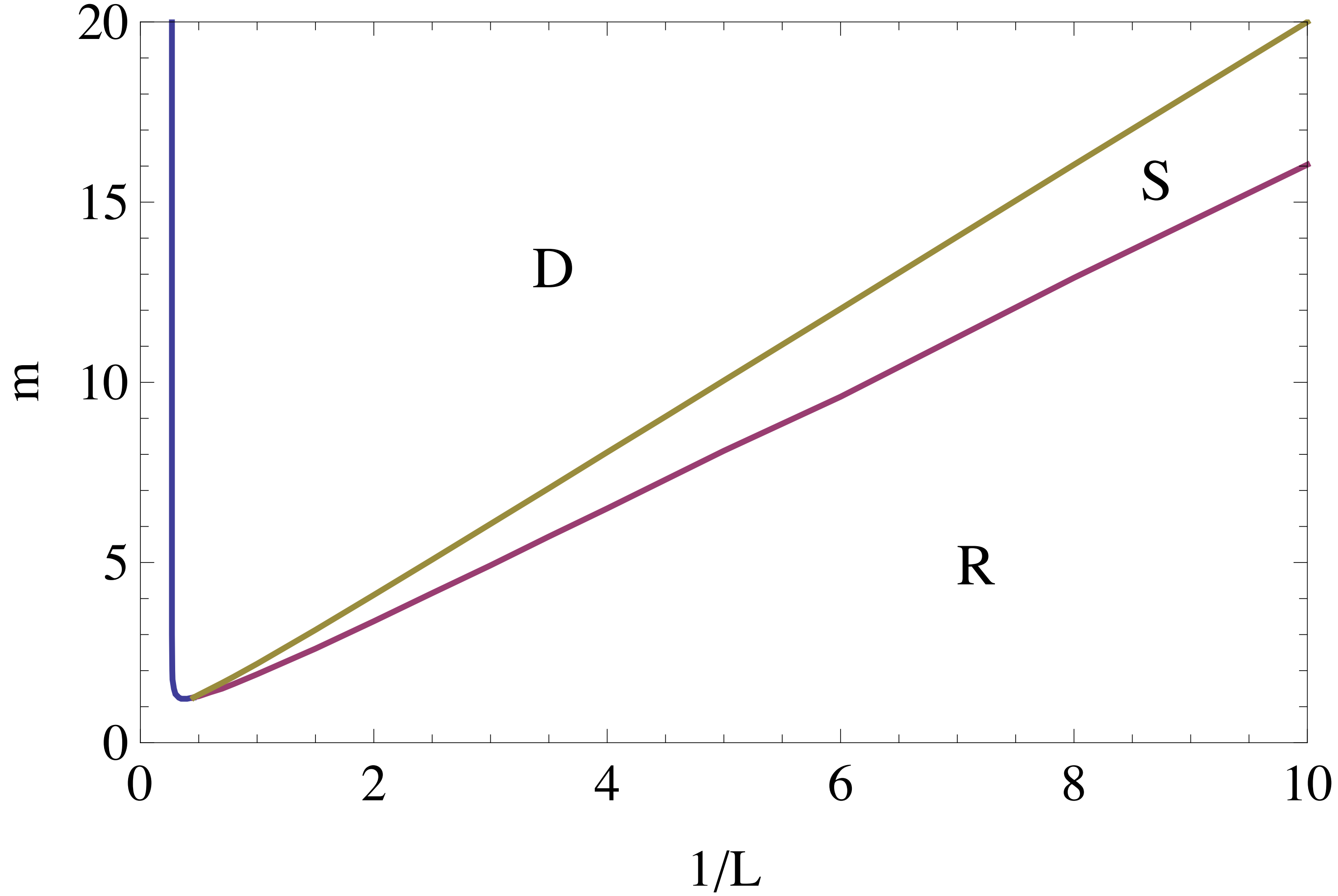
$\langle A \rangle = 0$ on the lattice

$\langle P \rangle \neq 0$ on the lattice

- DGSB by Hosotani mechanism is topological phenomenon.
- Can be indirectly observed from Gauge-invariant quantity.

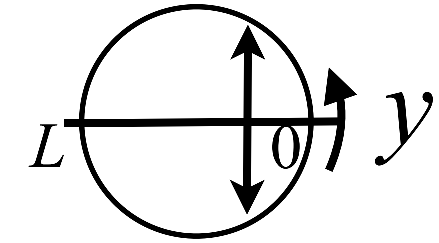
SU(3) adj. with non-perturbative deformation

$$\mathcal{V}_g^{\rm np} = -\frac{2}{L^4\pi^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{\cos(2n\pi q^{ij})}{n^4} + \frac{M^2}{2\pi^2 L^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{\cos(2n\pi q^{ij})}{n^2}$$



Orbifold and chiral fermions

S^1/Z_2



orbifold B.C.

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}_{x,-y} = P_0 \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}_{x,y} P_0^\dagger$$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}_{x,L-y} = P_1 \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}_{x,L+y} P_1^\dagger$$

Symmetry breaking by (P_0, P_1)

Chiral fermion

$$\psi(x,-y) = P_0\gamma_5\psi(x,y)$$

$$\psi(x,L-y) = P_1\gamma_5\psi(x,L+y)$$

EW theory

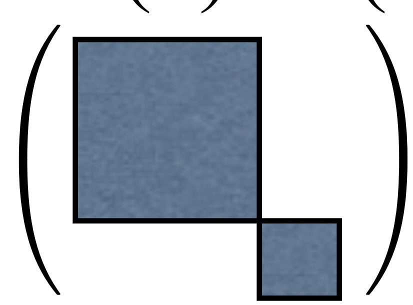
$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

Higgs: SU(2) doublet $\in A_y$

$$G \underset{(P_0,P_1)}{\longrightarrow} SU(2)\times U(1) \underset{\theta_H \neq 0}{\longrightarrow} U(1)$$

e.g.)SU(3)

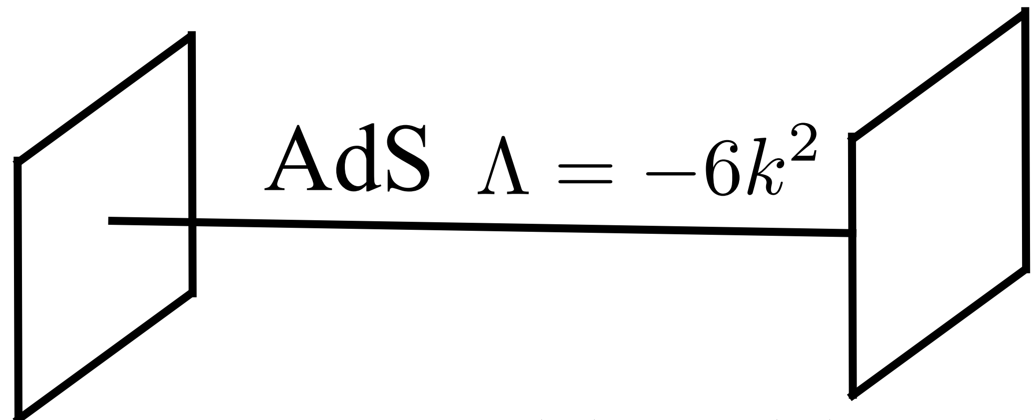
$$P_0 = P_1 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$A_\mu =$ 

$A_y =$ 

SO(5)×U(1) gauge-higgs unification in RS

Plank brane



TeV brane

Brane fermions
Brane scalars

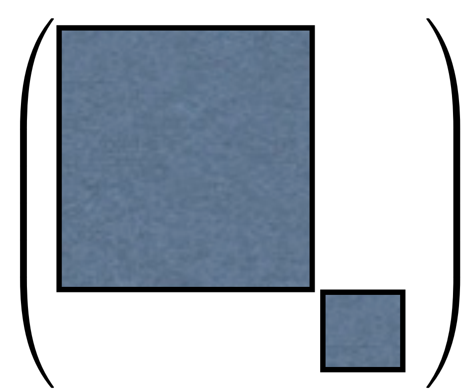
SO(5)×U(1)

W, Z, γ

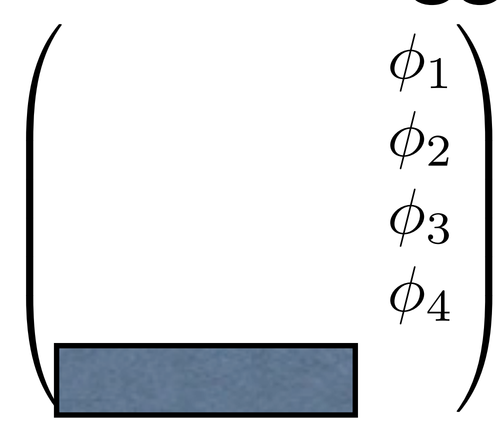
Higgs

$$P_0 = P_1 = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & +1 \end{pmatrix}$$

$A_\mu =$



$A_y =$



(P_0, P_1) SO(5) $\to$ SO(4)~SU(2)×SU(2)

Brane scalars SO(4)×U(1) $\to$ SU(2)×U(1)

$\theta_H \neq 0$ $\to$ U(1)